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Recipe for Calculating the Pulse Response of Dipole Antennas

Dieter Lohrmann

Surface EW Systems Branch
Tactical Electronic Warfare Division

Jack Bernardes

Naval Surface Warfare Center
Dahlgren, Virginia

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Dieter Lohrmann and Jack Bernardes*

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Naval Research Laboratory
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This report describes a simplified method for calculating the impulse performance of electrically long dipoles. An equivalent circuit for approximating the input impedance is derived, permitting complex impedances of the driving pulse source to be treated. MATLAB programs for the calculations are added. An experiment is described in which the short circuit output current of a charged monopole (half dipole) was oscillographed and compared with the calculated results.

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RECIPE FOR CALCULATING THE PULSE RESPONSE OF DIPOLE ANTENNAS

BACKGROUND

There has recently been considerable interest in the pulse performance of dipole antennas, particularly in connection with high power microwave (HPM) work. In this work, a capacitive source is discharged into a dipole. Traditionally, this problem has been treated theoretically by using known steady state periodic solutions in the frequency domain and calculating the real-time pulse response of the dipole by Laplace Transform. Although this method gives technically useful solutions, it falls short in explaining certain effects seen in experiments. The reason is that the solutions in the frequency domain do not include the frequencies at zero and infinity. An example of this is a battery connected to a long dipole. "Long," in this context, means that the rise time of the battery current is short in comparison to the time it takes the wave to travel to the end of the antenna, get reflected, and return to the source.

The work by O. Einarsson [1] and T.T. Wu [2] give the exact solution for the current on a long cylinder-type dipole antenna resulting from a voltage step at the input, using Maxwell's Equations.

Several features of these solutions are remarkable in that they are not intuitively obvious:

1. At the first moment after application of the voltage step, the antenna exhibits a zero input impedance, i.e., the current is infinite.

2. The current decays with a time constant that depends only on the radius a of the antenna cylinder and, of course, not on the length of the antenna. The decay time decreases with decreasing diameter.

3. After a time which is large in comparison to a/c, with c being the speed of light, the current at the input decays proportional to $1/\log(t)$.

4. The current waveform created at the input travels down the antenna with the speed of light, without any change of shape. This is remarkable because one might expect the current waveform to be attenuated due to radiation loss while traveling along the dipole, but such is not the case. Figure 1 shows the current at the input of the antenna following the application of a 2 V step at the input [1]. Reference 1 also provides a detailed table of the current vs normalized time. The ordinate in Fig. 1 shows the current at the input in amperes, the abscissa shows the normalized time

$$v = (1/a) * \sqrt{(c*t)^2 - z^2}\,, \tag{1}$$

where a is the radius of the dipole cylinder, c is the speed of light, and z is the length coordinate on the dipole, starting at the feed point. Note that at the feed point, $z = 0$ and $v = c/a * t$.

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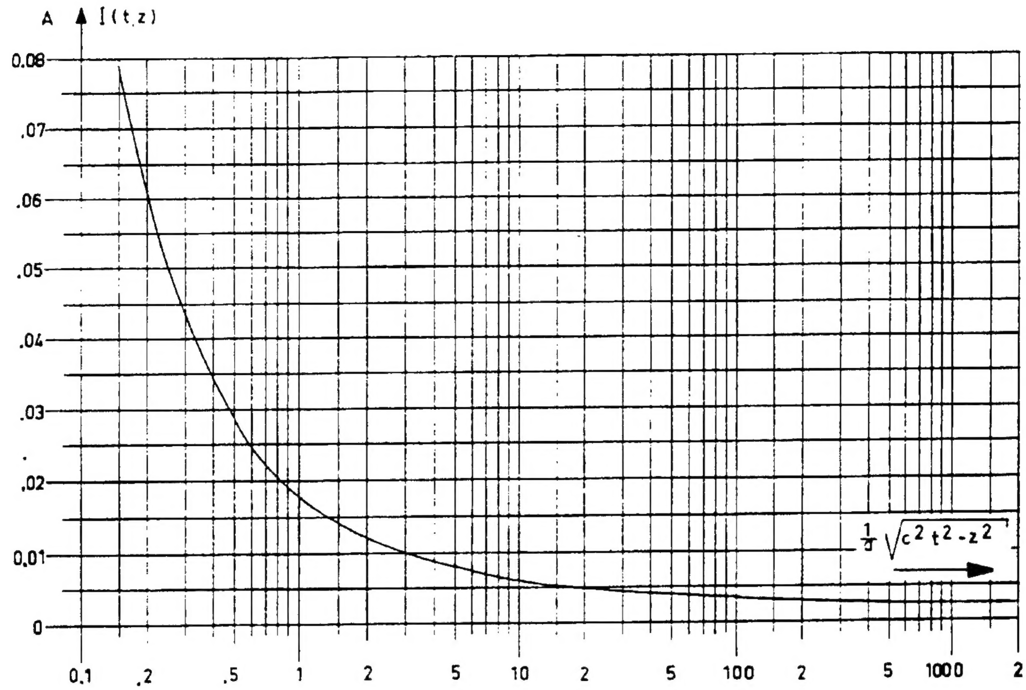

$I(t, z)$ as a function of $\frac{1}{a}\sqrt{c^2t^2-z^2}$.

Fig. 1 — Current into dipole, following a 2 V voltage step input vs normalized time, from Ref. 1

MODELING THE DIPOLE

The exact solution for the current is very complicated. Therefore, it is not well suited for solving practical problems, like taking into account the impedance of the source. For this reason, an attempt was made to model the input impedance of the antenna, simulating the accurate performance as closely as possible.

The antenna, in a way, acts as a transmission line. However, since the current is infinite at time $t = 0$ for a voltage step input with zero rise time, the input impedance of the antenna must be zero for infinite frequency. A transmission line with distributed series R and L and shunt G and C has this property only if the distributed inductance is zero. Therefore, the input impedance is modeled as

$$Z(j\omega) = \sqrt{R/(G + j\omega C)}\ . \tag{2}$$

In the following, instead of a dipole, we consider a monopole over a conducting plane. The input voltage is now a 1 V step instead of 2 V as in Ref. 1, the current is the same, and Z is the input impedance of the monopole. For a dipole, the input impedance is then $2Z$.

Next, we investigate how accurately the current resulting from a voltage step applied to Z will approximate the exact solution. To this purpose, the current vs time response is calculated by Inverse Laplace Transform and compared to the exact solution shown in Fig. 1, using the model of Eq. (2):

$$I(s) = V(s)/Z(s)\ , \tag{3}$$

$V(s) = Vz/s$ is the voltage step in the frequency domain, and Vz is the amplitude of the step. Then,

$$I(s) = Vz/s/Z(s) = Vz/\left(s * \sqrt{R/(G+s*C)}\right) \ , \tag{4}$$

and

$$I(t) = 1/(2*\pi j) \int_{s=-j\infty}^{s=j\infty} I(s)\exp(s*t)ds. \tag{5}$$

The solution of this integral is given in the correspondence tables [3]:

$$I(t) = \sqrt{C/R}\left\{1/\sqrt{\pi*t}*\exp(-b*t)+\sqrt{b}*\mathrm{erf}\left(\sqrt{b*t}\right)\right\}. \tag{6}$$

Here, $b = G/C$, and erf is the Gaussian error function

$$\mathrm{erf}(x) = 2/\sqrt{\pi} * \int_{\xi=0}^{\xi=x} \exp(-\xi^2)d\xi. \tag{7}$$

Replacing t by v and setting the length coordinate $z = 0$: $v = c/a * t$, $t = v * a/c$.

Letting $p = \sqrt{(C*c)/(\pi*R*a)}$, $\quad q = a*b/c$:

$$I(v) = p*\left\{1/\sqrt{v}*\exp(-q*v)+\sqrt{\pi*q}*\mathrm{erf}\left(\sqrt{q*v}\right)\right\}. \tag{8}$$

The coefficients p and q are chosen such that $I(v)$ resembles the exact function of the current shown in Fig. 1 as closely as possible. However, for practical purposes, it is sufficient to limit this range to $1 \le v \le 100$. $v \ge 1$ means that we consider only values of t that are greater than or equal to the time it takes light to travel the radius of the antenna tube. This appears to be a reasonable restriction because, in practice, leads connecting the source to the antenna will not be shorter than the radius a of the antenna cylinder. Furthermore, $v \le 100$ means that only times are considered that are smaller than or equal to the time it takes light to travel 100 radii. Depending on the particular application, the values of p and q can be chosen to emulate the curve for different regions of v. For the range chosen here, for $1 \le v \le 100$

$$p = 0.01799 \text{ and } q = 0.01094 .$$

With these values, Fig. 2 shows the approximated curve and the exact curve. The error of the approximation is less than 10%.

Now one can express $Z(j\omega)$ using the parameters of the dipole:

$$Z(j\omega) = 1/p*\sqrt{c/(a*\pi)}\ \sqrt{1/(q*c/a+j\omega)} \ . \tag{9}$$

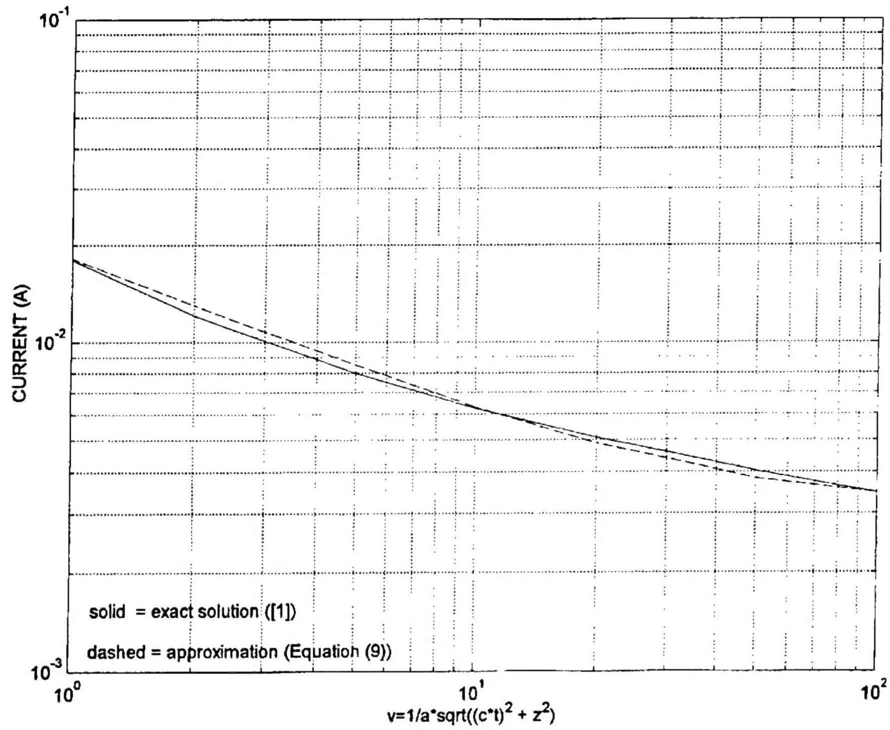

Fig. 2 — Current response to input voltage step of one volt to monopole exact solution and approximation

This can be rewritten as

$$Z(j\omega) = 31.4\Omega * \sqrt{1/(0.01094 + a/c * j\omega} \qquad \text{for the monopole,} \tag{9a}$$

and

$$Z(j\omega) = 62.8\Omega * \sqrt{1/(0.01094 + a/c * j\omega} \qquad \text{for the dipole.} \tag{9b}$$

This is the approximated pulse input impedance of the dipole and the monopole, where $2a$ is the diameter of the antenna cylinder and c is the speed of light. This is valid for time $a/c \leq t \leq 100\ a/c$ and until the wave reflected at the end of the antenna reaches the feed point.

Now the current to the monopole can be calculated for any given source impedance. However, since the source impedance can be a rather complicated function, the Inverse Laplace Transform correspondence may not be given in any table, and therefore, numerical integration may be necessary. To show the procedure and the problems associated with this, $I(t)$ derived above is now calculated by numerical integration of Eq. (5) by using Eqs. (3) and (9). The result must be the same as shown in Fig. 2, if everything goes well.

Clearly, we cannot numerically integrate to infinity. Hence, $s = \mu + j\omega$ must be terminated at some value, but at which?

First, we choose the path of integration in the right half of the complex plane, leaving all poles to the left of the integration path, see Fig. 3. The path is chosen to go parallel to the imaginary axis at distance μ, with $j\omega$ the parameter.

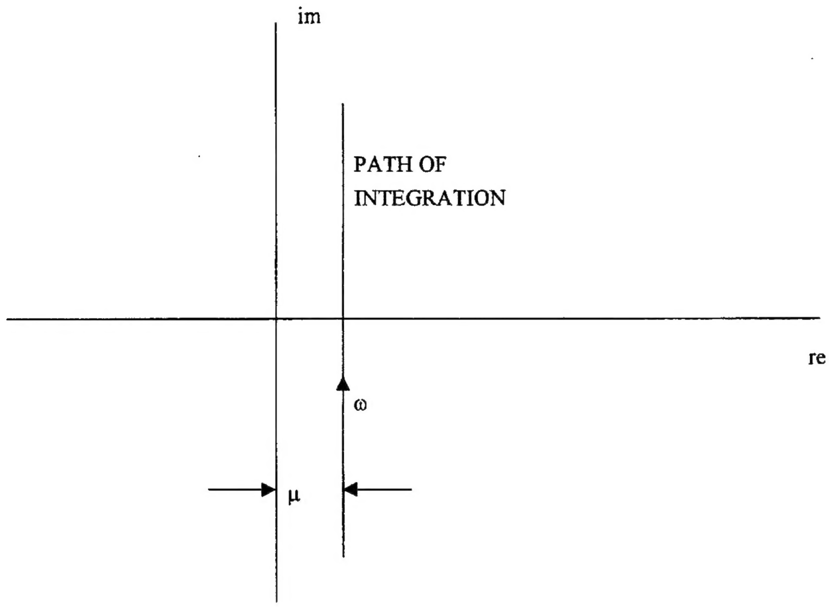

Fig. 3 — Path of integration on complex plane

It is a good idea to see what the function to be integrated looks like. It can be shown that the real part of the function of ω under the integral is odd in regard to the integration parameter ω. Therefore, by integration from minus omega to plus omega, the imaginary part of Eq. (5) is zero. This makes sense because the solution of the real time current cannot be complex. Therefore, we need to consider only the real part of Eq. (5). Furthermore, because the real part of the function under the integral is an even function of ω, we need to integrate only from $\omega = 0$ to $\omega = \omega_{max}$ and multiply the result by a factor two.

Replacing *s* in Eqs. (4) and (5) by $\mu + j\omega$, *ds* by $jd\omega$ (μ is constant on the path), and *t* by $v * a/c$, one finds:

$$I(t) = 4 * \sqrt{\pi} / p * \exp(\mu * v) \int_{\omega=0}^{\omega=\omega_{max}} ((\mu + q)^2 + \omega^2))^{1/4} / (\mu^2 + \omega^2)^{1/2}$$
$$* \cos(\omega * v + 1/2 * \arctan(\omega/(\mu + q)) - \arctan(\omega/\mu)) d\omega \; . \tag{10}$$

The function under the integral of ω is plotted in Fig. 4 with $\mu = 1/v$. For large omega, the function oscillates with a period close to $2 * \pi$, but the amplitude decays very slowly. Therefore, integration to very high values of omega would be required, which in turn would require excessive computation time. However, for large omega, the function is nearly periodic in

$\omega * v = k * 2 * \pi$, with *k* being an integer.

A simple method to obtain a good approximation is to evaluate the integral first to a multiple of $k * \pi$, then to $(k + 1) * \pi$, add both values, and divide the result by two. With $\mu = 1/v$, a value of $k = 50 \ldots 100$ renders values that are accurate to a few percent.

SELECTION OF μ

Theoretically, μ can be chosen to be any real value greater than zero. However, because the factor $\exp(\mu * v)$ appears in front of the integral in Eq. (10), that factor can become excessively large, which may

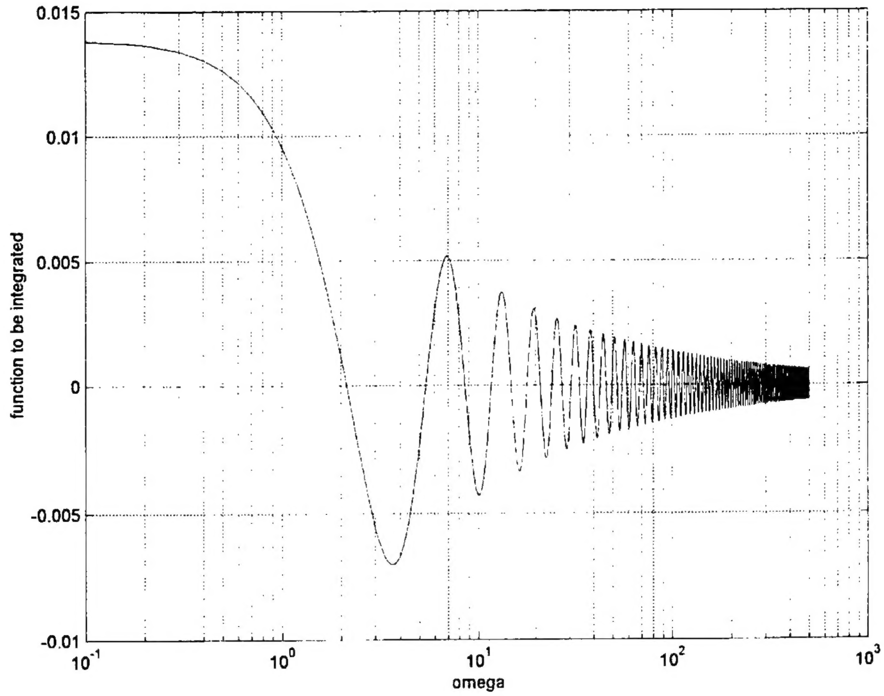

Fig. 4 — Real part of the function under the integral in Eq. (11) vs ω

cause problems in the numerical integration. Good values are obtained by choosing m such that $1/(m * \nu)$ $\exp(m * \nu)$ becomes a minimum, which is the case for $m * \nu = 1$, or $m = 1/\nu$.

In practice, the integral need not be programmed for integration by machine in the form of Eq. (10). Rather, it can be processed in the simpler form of Eq. (11), provided that the machine can process complex arithmetic:

$$I(t) = \mathrm{Re}\left\{ 1/\pi \int_{\omega=0}^{\omega=\omega_{max}} I(\mu + j\omega) * \exp(\mu + j\omega) * t \, d\omega \right\}, \quad (11)$$

with $I(\mu + j\omega) = Vz/(\mu + j\omega)/Z(\mu + j\omega)$, and

$$Z(\mu + j\omega) = 1/p * \sqrt{c/(a*\pi)} * \sqrt{1/(q*c/a + \mu + j\omega)} \; .$$

Letting $\mu = 1/t$, and $\omega_{max} = k * \pi/t$, with $k = 50\ldots100$, gives a satisfactory solution.

TAKING THE SOURCE IMPEDANCE INTO ACCOUNT

Figure 5 shows the arrangement,

$$I(s) = Vz/s/(\Re(s) + Z(s)) \, , \quad (12)$$

where $\Re(s)$ is the impedance of the source and $Z(s)$ is the impedance of the monopole, as given in Eqs. (9), (9a), and (9b). Again, s is replaced in Eq. (12) by $s = \mu + j\omega$ and inserted into Eq. (11).

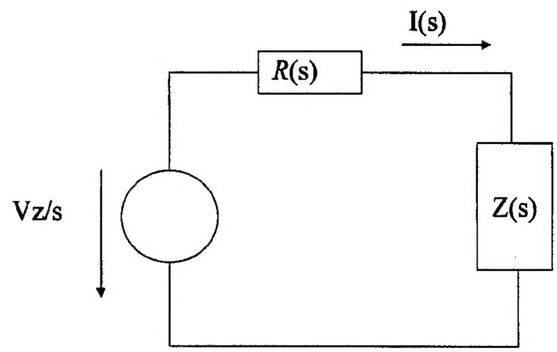

Fig. 5 — Step voltage source with source impedance $R(s)$

Example

In an experiment, the monopole antenna consisted of a 20.3-cm-diameter cylinder ($a = 0.1015$ m), 116-cm long, raised 3.8 cm above a conducting ground plane. A high-voltage step transition was applied to the antenna input, and the resulting input current was measured. The antenna tube was positioned vertically over a brass-screen ground plane that was housed in a 12.2-m wide, 30.5-m long metal-skin building. Two dielectric angled braces kept the tube vertical (Fig. 6). The base of the tube was separated from the ground plane by a small spark gap and a current-viewing resistor (CVR), as shown in Fig. 7. One spark gap electrode was connected to the ground plane and the other was connected to the tube base through a coaxial CVR (T&M Research SBNC-1-05). The CVR measured the current between the top electrode and the tube. During operation, the tube was charged to 17.5 kV with respect to the ground plane by a resistively isolated power supply. When the spark gap broke down, the desired voltage transition (negative-going step function) was generated. The spark gap operated in a stable, repetitive, self-breakdown mode with a gap spacing of 1.9 mm, at an operating pressure of 2 atm (30 psi).

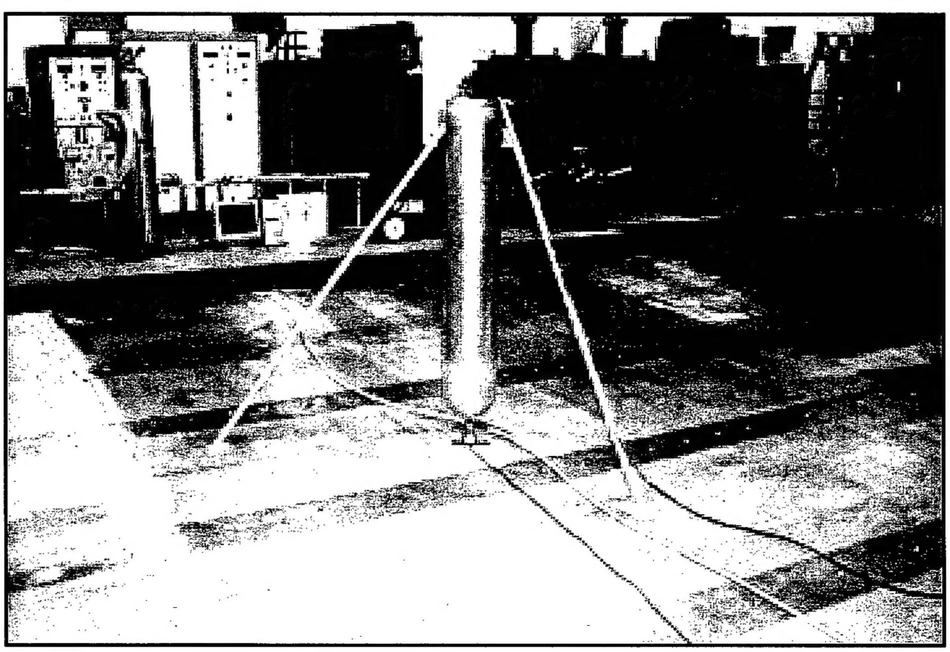

Fig. 6 — View of monopole-antenna test setup

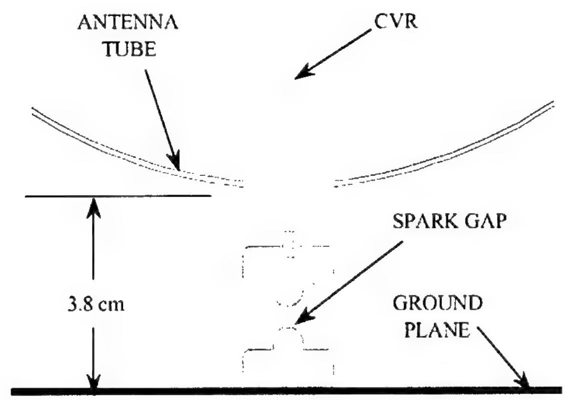

Fig. 7 — Cross-sectional view of the monopole antenna feed region

A high-bandwidth fiber-optic link (Nanofast OP 300-2A) transmitted the measured input current to a digitizing oscilloscope (LeCroy LC574A). The battery-powered receiver portion of the link was located inside the monopole tube, and it relayed the measured current via a dielectric fiber-optic cable to a receiver unit co-located in a shielded enclosure with the oscilloscope. The bandwidth of this measurement system was 1 GHz. The measured waveform represents an average of 100 pulses. The initial 10 to 90% risetime is approximately 2 ns, and the period of the oscillations is approximately 20 ns. The risetime is mainly controlled by the spark-gap turn-on time, and the period is related to the four-way transient time of the induced voltage transient between the antenna input and the end of the tube.

The source provided a voltage step of $Vz = 17.5$ kV with a source impedance of 50 nH. Hence, $\Re(j\omega) = j\omega L$, with $L = 50$ nH. For $v = c * t/a = 1$, $t = v * a/c = 0.34$ ns. The wave returns to the feed point after 7.7 ns, i.e., $v = 23$, which is fulfilling the requirement that it be smaller than 100.

Figure 8 shows the input current of the monopole vs time, calculated using the model. During the first three nanoseconds, the current rises to 284 A and then settles to a slowly decaying level of about 70 A. This waveform would be seen at the input of the monopole until the wave reflected at the end of the antenna arrives back at the feed point. Because the length of the monopole was 116 cm, this would occur after 7.7 ns.

COMPARISON WITH MEASURED RESULTS

Figure 9 compares the calculated with the measured results for the time interval from $t = 0$ to $t = 8$ ns. The calculated peak is considerably higher than the measured one. The voltage step of the source was 17.5 kV, the source impedance was assumed to be an inductor of 50 nH and an inductor of 50 nH with a resistor of 50 Ω in series. The latter matches the measured results quite well. The measured current, however, starts with a horizontal tangent; this may be due to the transient resistance characteristic of the spark gap. This characteristic is time- and current-dependent and nonlinear; it was not included in the model. The geometry of the current path through the spark gap and CVR determines the source inductance. Source resistance is an approximation of the average arc resistance during the period of interest.

VOLTAGE RESPONSE TO A CURRENT STEP

Next we can calculate the voltage response at the input to the monopole resulting from a current step. Let the current step Iz be

$$I(s) = Iz/s.$$

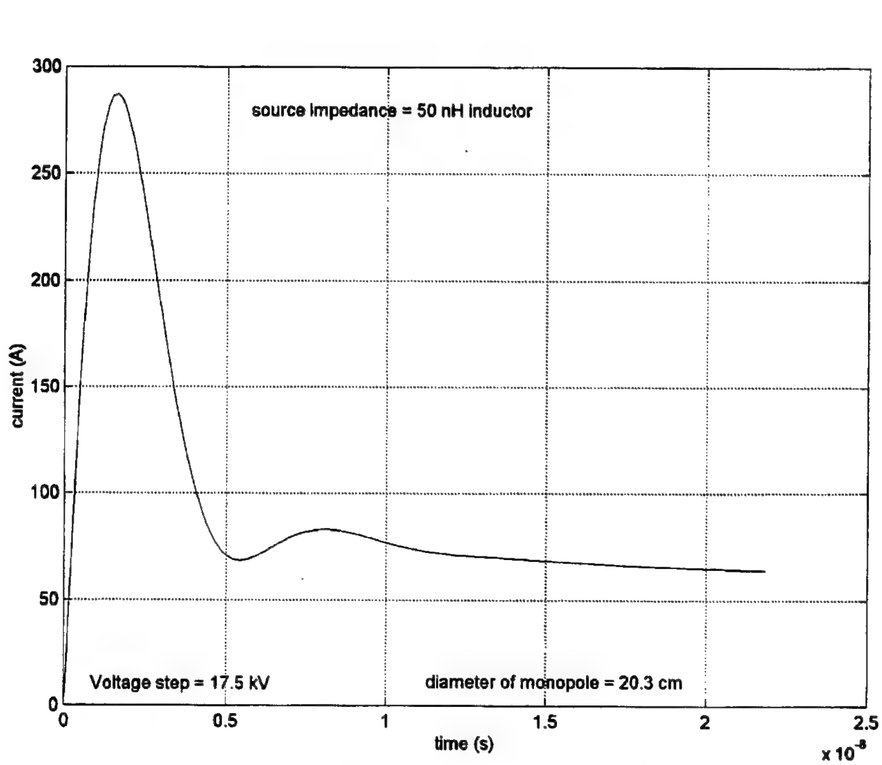

Fig. 8 — Input current to monopole with 50 nH source impedance

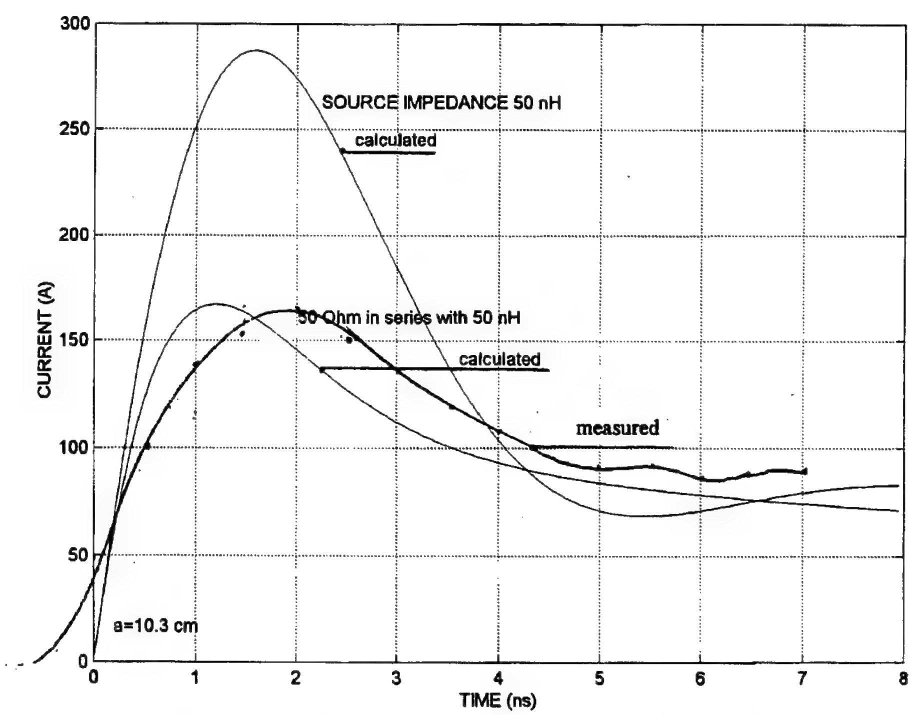

Fig. 9 — Comparison of measured and calculated results using the model

Then,

$$V(s) = Iz/s * Z(s)\,, \tag{13}$$

and

$$V(t) = \mathrm{Re}\left\{ 1/\pi * Iz * \int_{\omega=0}^{\omega=\omega_{max}} Z(\mu + j\omega) * \exp((\mu + j\omega) * t) / (\mu + j\omega) d\omega \right\}. \tag{14}$$

The solution of this integral is given in closed form in the Laplace transform correspondence table [3]; it is

$$V(t) = Iz\left(p * \sqrt{\pi * q}\right) * \mathrm{erf}\left(\sqrt{q * v}\right), \qquad \text{with } v = c/a * t. \tag{15}$$

Figure 10 is a plot of V as function of the normalized time $v = c/a * t$ for a current step of 1 A. Numerical integration using Eq. (11) gives the same result.

Again, it is seen that the input impedance V/I at the first moment is 0, i.e., the monopole exhibits a short circuit at the beginning. The voltage then rises with a tangent perpendicular to the abscissa, similar to $\sqrt{t}$. The rise time is proportional to the diameter of the monopole cylinder. For the time going to infinity, the voltage also tends toward infinity, i.e., there is no finite asymptote.

It has been suggested that the current pole at $t = 0$ for a step voltage input could be explained by capacitive effects at the feed point of the antenna. But this does not seem to be the case, because calculation of the input voltage following a current step by using the solution of Maxwells Equations given in Ref. 4 yielded the same result [5]. In Ref. 5, it was assumed a priori that the current wave moves along the dipole

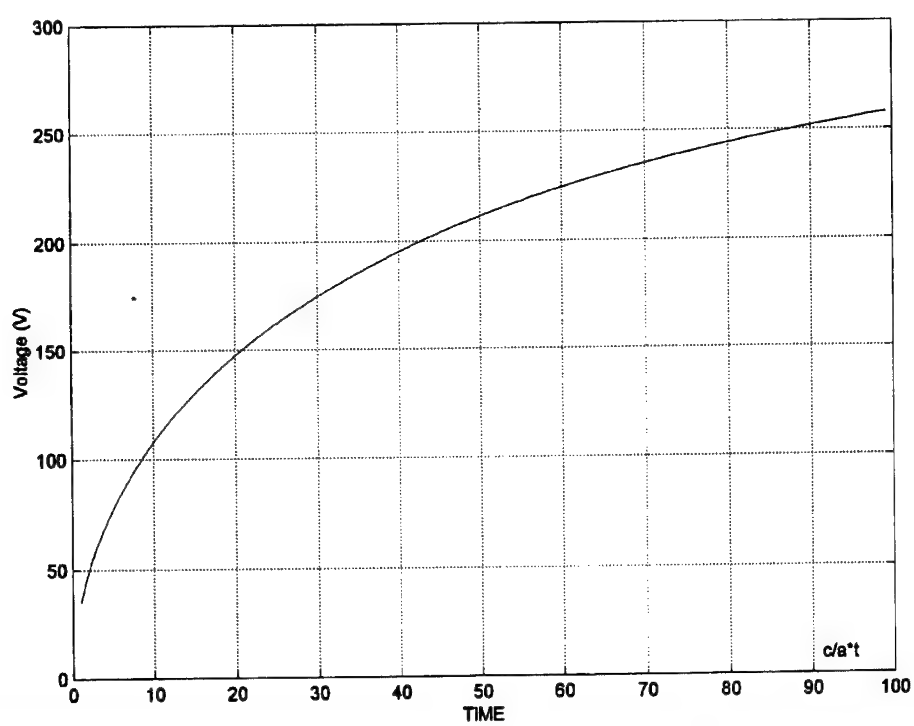

Fig. 10 — Voltage at monopole input following a 1 A current step

unabated, with the speed of light. Since only the current appears in the equations, and not the electric fields between the dipole, it follows that this is not an effect caused by capacity at the feed point, but is inherent to the propagating magnetic field.

RECIPE TO CALCULATE THE PULSE INPUT CURRENT OF A DIPOLE

Given: Source providing voltage step Vz, having source impedance $\Re(j\omega)$.
Wanted: current to dipole $I(t)$.
For values of time t, $I(t)$ may be calculated by numerical integration using the following equation:

$$I(t) = \mathrm{Re}\left\{ Vz/\pi \int_{\omega=0}^{\omega=\omega_{max}} \exp(w*t)/((\Re(w)+Z(w))*w)d\omega \right\}, \tag{16}$$

$1 \le c/a * t \le 100$, and $c * t < 2 * le$, with $w = 1/t + j * \omega$,

$$Z(w) = 31.4\Omega * \sqrt{1/(0.01094 * c/a + w)}\,, \quad \text{for monopole,} \tag{17}$$

and

$$Z(w) = 62.8.\Omega * \sqrt{1/(0.01094 * c/a + w)}\,, \quad \text{for dipole,} \tag{18}$$

and $2a$ is the diameter of the dipole, c is the speed of light, le is the length of the monopole equal to one-half length of the dipole.

Integrate to $\omega_{max} = 100 * \pi/t$, then again to $\omega_{max} = 101 * \pi/t$, add both values, and divide the result by 2. The result is the desired current $I(t)$. Standard available programs for quadrature may be used for complex integration.

The solution is valid until the wave reflected from the end of the antenna returns to the source.

Typically, 100 values of $I(t)$ are calculated per minute using MATLAB on a PC.

REFLECTIONS AT THE OPEN END OF THE MONOPOLE

Calculation Using a 50 Ω Transmission Line

So far, we have dealt only with what happens during the time the wave travels forward on the monopole or dipole, respectively. Next we calculate the input current at the time after the signal reflected at the open end of the monopole appears at the feed point. To demonstrate the method, we first calculate the input current of an open 1.2-m long transmission line, after a 50 V voltage step was applied thru a 50 nH inductor. Figure 11 shows the setup.

At $t = 0$, the voltage Vz = const. = 50 V is switched onto a transmission line. The characteristic impedance Z of the line is constant = 50 Ω; the electrical length le is 1.2 m, i.e., the same as the length of the monopole, and the line is open at the end.

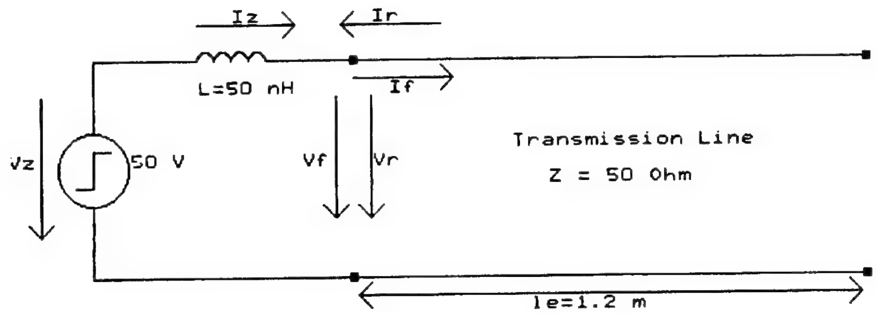

Fig. 11 — Voltage step to transmission line

At $t = 0$, a current wave $If1 = Vz/Z$ starts traveling forward on the line with speed of light c. Also, a voltage wave $Vf1$ with amplitude Vz starts traveling forward. In the following, Vf and If refer to the forward-going voltage and current waves at the input of the line. Vr and Ir are the voltage and current waves traveling on the line in reverse direction. The number 1 denotes the state at the input during the time $0 \le t < 2 * le/c$, the number 2 is the state during the time interval $2 * le/c \le t < 4 * le/c\ldots$.

The source has an internal inductance $L = 50$ nH. The voltage drop across L is VL. The circuit equations in the time domain yield:

$$Vz = Vf + Vr + VL \tag{19}$$

$$Iz = If - Ir \tag{20}$$

$$Vf/If = Z \tag{21}$$

$$Vr/Ir = Z \tag{22}$$

$$VL = L * dIz/dt. \tag{23}$$

The circuit equations in the frequency domain are:

$$Vz/s = vf(s) + vr(s) + vl(s) \tag{19a}$$

$$iz(s) = if(s) - ir(s) \tag{20a}$$

$$vf/if = Z \tag{21a}$$

$$vr(s)/ir(s) = Z \tag{22a}$$

$$vl(s) = L * s * iz(s) - L * Ilz(t) \tag{23a}$$

where

$Ilz(t = 0) = Ilz1$ is the current in the inductor at time $t = 0$, and
$Ilz(t = 2 * le/c) = Ilz2$ = current in the inductor at $t = 2 * le/c$, i.e., when the wave returns to the source after the first reflection at the open end of the line.

For the time interval #1 until the wave reflected at the open end of the line returns to the feed point:

$$0 < t < 2 * le/c \quad vr1 = 0, ir1 = 0,$$

with Eq. (20a):

$$\begin{aligned} iz1 &= if1; \\ vf1 &= if1 * Z = iz1 * Z; \end{aligned} \tag{24}$$

with Eq. (19a):

$$Vz/s = iz1 * Z + s * L * iz1 - L * Ilz1; \tag{25}$$

solving for $iz1$:

$$iz1 = (Vz/s + L * Ilz1)/(Z + s * L). \tag{26}$$

This is the input current to the line in the frequency domain for $0 \le t < 2 * le/c$. The current in the time domain is then

$$Iz1(t) = 1/(2 * \pi * j) * \int_{s=-j\infty}^{s=j\infty} iz1 * \exp(s * t) ds. \tag{27}$$

At $t = le/c$ the wave arrives at the open end of the line and is reflected, traveling in reverse direction. The current undergoes a phase reversal. This has already been taken into account by pointing the reverse current arrow in reverse direction. Hence, at the time when the reflected current $ir1$ arrives at the feed point:

$$ir2 = vf1 * \exp(-2 * le * s/c). \tag{28}$$

The voltage wave gets reflected at the open end in-phase:

$$vr2 = vf1 * \exp(-2 * le * s/c). \tag{29}$$

The factor $\exp(-2 * le * s/c)$ denotes the delay of $2 * le/c$ until the wave returns to the source.

Now the state for the time interval $2 * le/c \le t < 4 * le/c$:

At the time when the reflected wave returns at the source, the driving sources are the primary source Vz and the power in the reflected wave; the forward-going wave is adjusted such that Kirchhoff's laws are satisfied at the input of the line. Therefore:

$$if2 = iz2 + ir2 \tag{30}$$

$$vf2 = (iz2 + ir2) * Z, \tag{31}$$

with Eq. (19a):

$$Vz/s = (iz2 + ir2) * Z + vr2 + s * L * iz2 - L * Ilz2, \tag{32}$$

where $Ilz2$ is the current in the inductor at $t = 2 * le/c$. With $ir2 * Z = vr2$ and solving for $iz2$, one obtains

$$iz2 = (Vz/s - 2 * vr2 + L * Ilz2)/(Z + s * L), \tag{33}$$

where

$$vr2 = vf1 * \exp(-2 * le * s/c), \tag{34}$$

and $vf1$ is given in Eq. (24):

$$vf1 = (Vz/s + L * Ilz1)/(1 + s * L/Z), \tag{35}$$

$$Iz2(t) = 1/(2 * \pi * j) * \int_{s=-j\infty}^{s=j\infty} iz2 * \exp(s * t) ds. \tag{36}$$

$Iz1(t)$ and $Iz2(t)$ are calculated by numerical integration, using the program refl.m.

Figure 12 shows the calculated input current to the transmission line, with $Vz = 50$ V, $L = 50$ nH, $Z = 50$ Ω, and length of open line = 1.2 m.

The calculation was programmed in refl.m, using reflfct.m and reflfct2.m. In refl.m, first $Iz1(t)$ is calculated. This yields $Ilz2(t = 2 * le/c)$, which is then called and processed by reflfct2, which then is subsequently called by refl.m for final processing of the interval $2 * le/c \leq t < 4 * le/c$.

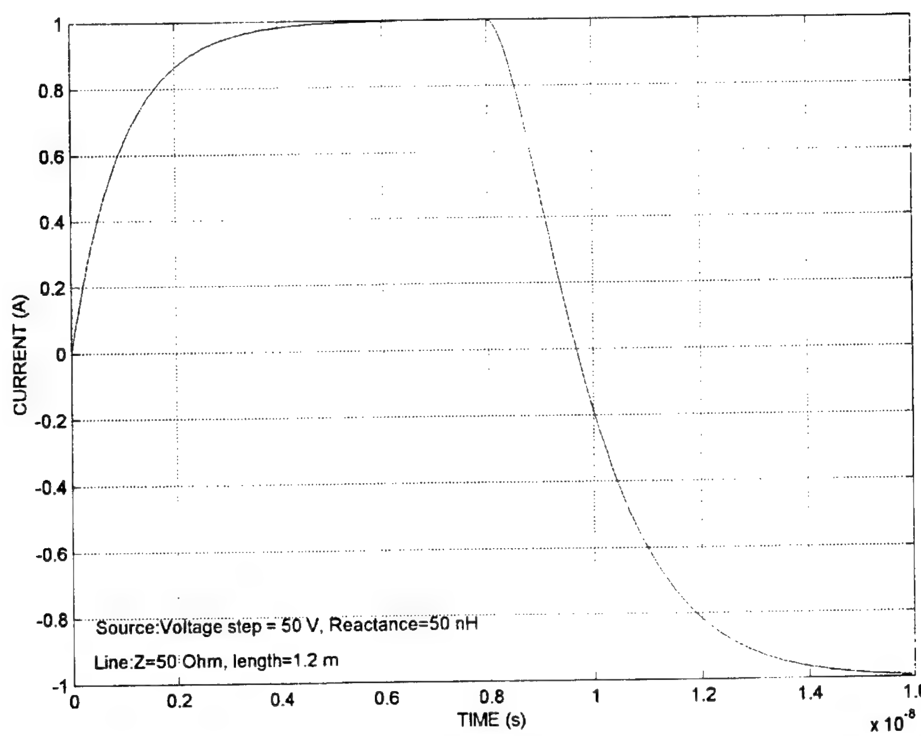

Fig. 12 — Input current to open transmission line

Reflections on Monopole

The calculations proceed analogous to the ones above, except that in Eqs. (21a), (22a), (24), (25), and (26), Z is replaced by

$$Z(s) = 31.4\Omega * \sqrt{1/(0.01094 + a * s/c)} .$$

Note that $Z(s)$ is **not** the characteristic impedance of the monopole. It is only used for replacement of Z in equations noted above. The exact solution of the voltage wave traveling along the monopole following a voltage step at the input is not known, and therefore, the characteristic impedance as a function of the length coordinate is not known either. However, a good approximation is to use the ratio of input voltage to input current at $t = 2 * le/c$. This value was used in the calculation for the value of Z in Eqs. (31), (32), and (33). The programs carrying out these calculations are refld.m with refldfc1.m and refldfc2.m, see appendix. The result is shown in Fig. 13. A loss of 15% of amplitude due to radiation at the open end of the antenna as seen in the experiment was included in the calculations. The theoretical exact solution of the mechanism of radiation at the end point of the monopole is not known at this time.

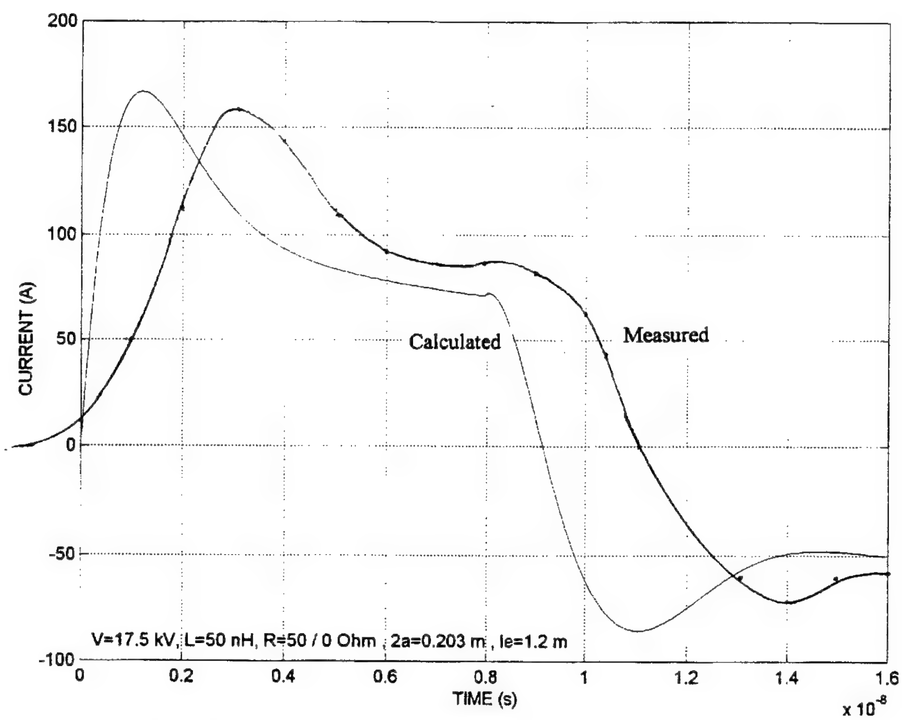

Fig. 13 — Input current to monopole with first reflection

A source resistance of 50 Ω in series to the 50 nH inductor was assumed for the first time interval before the reflection. This resistance represents the cumulative average resistance of the spark gap during establishment of the current. After the current is established, the resistance of the spark gap is very low. Therefore, for the reflected wave time interval, the series resistance of the source was neglected.

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Appendix
MATLAB PROGRAMS MENTIONED IN THE TEXT

```
% refld.m
% calculates input current to a monopole antenna
% line vs. time after a voltage step of 17500V. The
% source has a series inductor of 50 nH. The trans-
% antenna is 1.2 m long and has a diameter of 20.3 cm
%
% This program carries out the integration of the
% frequency domain function given by refldfc1.m
% and refldfc2.m
le=1.2;
c=3e8;
tmin=0.01e-9;
tmax=2*le/c;
ilz1=0;
nmax=150;
k=50;
n=1;
while n<nmax+1
   t=tmin+(tmax-tmin)*(n-1)/nmax;
   tt(n)=t;
   ommax1=t.\k*pi;
   ommax2=t.\(k+1)*pi;
   om=0:ommax1/1000:ommax1;
   refldfc1;
   z1=2*integral(y,0,ommax1);
   om=0:ommax2/1000:ommax2;
   refldfc1;
   z2=2*integral(y,0,ommax2);
   z(n)=real(z1+z2)/2;
   n=n+1
end
ilz2=z(n-1);
tmin=2*le/c;
tmax=4*le/c;
n=1;
while n<nmax+1
   t=tmin+(tmax-tmin)*(n-1)/nmax;
   tt(n+nmax)=t;
   ommax1=t.\k*pi;
   ommax2=t.\(k+1)*pi;
   om=0:ommax1/1000:ommax1;
   refldfc2;
   z1=2*integral(y,0,ommax1);
   om=0:ommax2/1000:ommax2;
   refldfc2;
   z2=2*integral(y,0,ommax2);
   z(n+nmax)=real(z1+z2)/2;
   n=n+1
   end
plot(tt,z)
grid
```

```
% refl.m
% calculates input current to a 50 Ohm transmission
% line vs. time after a voltage step of 50 V. The
% source has a series inductor of 50 nH. The trans-
% mission line is electrically 1.2 m long and open
% at the end.
% This program carries out the integration of the
% frequency domain function given by reflfct1.m
% and reflfct2.m
le=1.2;
c=3e8;
tmin=0.01e-9;
tmax=2*le/c;
ilz1=0;
nmax=150;
k=50;
n=1;
while n<nmax+1
   t=tmin+(tmax-tmin)*(n-1)/nmax;
   tt(n)=t;
   ommax1=t.\k*pi;
   ommax2=t.\(k+1)*pi;
   om=0:ommax1/1000:ommax1;
   reflfct1;
   z1=2*integral(y,0,ommax1);
   om=0:ommax2/1000:ommax2;
   reflfct1;
   z2=2*integral(y,0,ommax2);
   z(n)=real(z1+z2)/2;
   n=n+1
 end
 ilz2=z(n-1);
 tmin=2*le/c;
 tmax=4*le/c;
 n=1;
 while n<nmax+1
    t=tmin+(tmax-tmin)*(n-1)/nmax;
    tt(n+nmax)=t;
    ommax1=t.\k*pi;
    ommax2=t.\(k+1)*pi;
    om=0:ommax1/1000:ommax1;
    reflfct2;
    z1=2*integral(y,0,ommax1);
    om=0:ommax2/1000:ommax2;
    reflfct2;
    z2=2*integral(y,0,ommax2);
    z(n+nmax)=real(z1+z2)/2;
    n=n+1
    end
  plot(tt,z)
  grid
```

```
% program name refldfc1.m
% contains the frequency domain function to be
% integrated by refld.m
le=1.2;
a=0.1015;
%vz=17500;
vz=5e5;
%l=50e-9;
l=4e-9;
%rl=50;
rl=0;
c=3e8;
ilzl=0;
s=t.\l+j*om;
zz=31.4*sqrt((0.01094+s*a/c).\l);
y=(s.*(1+zz.\(s*l+rl))).\(zz.\vz+zz.\s*l*ilzl).*exp(s*t)/2/pi;

% program name refldfc2.m
% contains the frequency domain function to be
% integrated by refld.m, 2*le/c<t<4*le/c
le=1.2;
%vz=17500;
vz=5e5;
%l=50e-9;
l=4e-9;
%rl=0;
rl=0;
c=3e8;
att=0.85;
zal=240;
a=0.1015;
s=t.\l+j*om;
zz=31.4*sqrt((0.01094+s*a/c).\l);
vr2=(1+zz.\(s*l+rl)).\((s.\vz+l*ilzl).*exp(-2*le*s/c));
y=(zal+(s*l+rl)).\(s.\vz-2*att*vr2+l*ilz2).*exp(s*t)/2/pi;
```

```
% program name reflfct1.m
% contains the frequency domain function to be
% integrated by refl.m
le=1.2;
vz=50;
l=50e-9;
c=3e8;
ilzl=0;
zz=50;
s=t.\1+j*om;
y=(s.*(1+s*l/zz)).\(vz/zz+s*l*ilzl/zz).*exp(s*t)/2/pi;

% program name reflfct2.m
% contains the frequency domain function to be
% integrated by refl.m, 2*le/c<t<4*le/c
le=1.2;
vz=50;
l=50e-9;
c=3e8;
zz=50;
s=t.\1+j*om;
vr2=(1+s*l/zz).\((s.\vz+l*ilzl).*exp(-2*le*s/c));
y=(zz+s*l).\(s.\vz-2*vr2+l*ilz2).*exp(s*t)/2/pi;

% program name integral.m
% This function 'integral'  uses Simpson's Rule
% for integration of a set of given values of a function
% y(x). The function values are assumed to be spaced
% in equal increments between a start value x1 and an
% end value xn. This has the advantage over Matlab's
% 'quad' function function that 'integral' can pick up
% quantities given in the work space.
function z=integral(y,x1,xn);
l=length(y);
h=(xn-x1)/(l-1);
s1=0;
m=1;
while m<(l-1)/2+1
   n1=2*m;
   s1=s1+y(n1);
   m=m+1;
end;
s1=4*s1;
s2=0;
m=1;
while m<(l-1)/2
   n2=2*m+1;
   s2=s2+y(n2);
   m=m+1;
end;
s2=s2*2;
z=(s1+s2+y(1)+y(l))*h/3;
```